7. The asymptotic analysis has shown that the free vibrations of piezoceramic shells with preliminary polarization along the α_2-lines with frontal surfaces without electrodes can be subdivided into 1) quasitransverse with low variability $(0 \leqslant t < 1/_2)$, 2) quasitransverse with high variability $(1/_2 < t < 1)$, 3) quasitransverse with variability $t = 1/_2$, 4) quasitangential $(0 < t < 1)$, and 5) ultralow frequency of Rayleigh type $(0 \leqslant t < 1/_2)$.

Each of these types of vibrations is described by a corresponding set of equations. Such a classification is physically conceivable and considerably simplifies the calculation of the natural frequencies and the other desired quantities.

We note that for the same classification of the free vibrations as in the theory of non-electric shells, the systems of principal and additional boundary value problem equations differ qualitatively from the corresponding non-electric shell problems in the high order of the systems of equations, the large number of initial quantities, and the boundary conditions. Hence, the classification obtained for the free vibrations of piezoceramic shells should be considered as a generalization of the classification for the free vibrations of non-electric shells.

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DIFFERENTIAL GAMES WITH VARIABLE STRUCTURE, WITH A GROUP OF PURSUERS CHASING A SINGLE TARGET*

K.V. DEMIDOV

Differential games with variables structure /1/ is which m pursuers chase a single target are considered. Sufficient conditions are given for the pursuit problems in such games to be solvable. Strategies leading to capture are devised. An example of a game for which the sufficient conditions proposed are essential, is given.

Let the motion of the i-th object $(i = 1, \ldots, m)$ prior to switchover to be described by the following equation:

$$z_i^{\cdot(1)} = C_i^{(1)}(t)\, z_i^{(1)} + f_i^{(1)}(u_i^{(1)}) - g_i^{(1)}(v), \quad t \in (0, \tau_i) \tag{1}$$
$$z_i^{(1)}(0) = z_i^\circ$$

and after the switchover by

$$z_i^{\cdot(2)} = C_i^{(2)}(t)\, z_i^{(2)} + f_i^{(2)}(u_i^{(2)}) - g_i^{(2)}(v), \quad t \in (\tau_i, +\infty) \tag{2}$$
$$z_i^{(2)}(\tau_i) = B_i(\tau_i)\, z_i^{(1)}(\tau_i)$$

Here $z_i^{(1)} \in R^{n_i}$, $z_i^{(2)} \in R^{m_i}$, $C_i^{(1)}(t)$ and $C_i^{(2)}(t)$ are continuous $n_i \times n_i$ - and $m_i \times m_i$ matrices respectively, the matrix $B_i(t)$ is also continuous and of dimension $m_i \times n_i$, $u_i^{(j)} \in P_i^{(j)} \subset R^{r_i^{(j)}}$, $v \in Q \subset R^q$, Q, $P_i^{(j)}$ $(j = 1, 2)$ are non-empty convex compacta. The functions $f_i^{(j)}, g_i^{(j)}$ $(i = 1, \ldots, m;\ j = 1, 2)$ depend continuously on their arguments. We specify, in the Euclidean space R^{m_i}, the terminal sets $M_i = M_i^1 + M_i^2$, where M_i^1 is a linear subspace of R^{m_i}, M_i^2 is a convex compactum from the orthogonal complement L_i^1 to the subspace M_i^1.

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At every instant of time $t \geqslant 0$ the pursued has the vector $v \in Q$ at his disposal. The admissible control of the pursued is represented by an arbitrary, Lebesgue-measurable function $v(t), t \geqslant 0, v(t) \in Q$. Every i-th pursuer has at its disposal the vector $u_i^{(j)}$ $(j = 1, 2; i = 1, \ldots, m)$ and the i-th player chooses his controls from the class of quasistrategies, i.e. in the form $u_i^{(j)}(t) = U_i^{(j)}(t, z_i^\circ, v(t)), j = 1, 2$ (/2/). Here for every admissible control $v(t)$ of the pursued $u_i^{(j)}(t)$ is Lebesgue-measurable and for $t \in [0, +\infty)$, $u_i^{(j)}(t) \in P_i^{(j)}$ $(i = 1, \ldots, m; j = 1, 2)$. The i-th pursuer also determines the instant $\tau_i \in [0, +\infty)$ of switching the dynamics of motion of the i-th object from (1) to (2). Thus the admissible strategy of the i-th pursuer P_i will be represented by the triplet $(\tau_i, U_i^{(1)}, U_i^{(2)})$. We shall also assume that an additional constraint is imposed on the instant of switchover, namely, that the pursuers must switch over not later than at the instant θ, i.e. $\tau_i \leqslant \theta$ $(i = 1, \ldots, m)$.

The aim of the group of pursuers $P_1, \ldots, P_m$ is to bring, using the admissible strategies, at least one vector $z_i^{(2)}$ to the corresponding set M_i in some finite time T, no matter which admissible control $v(t)$ is chosen by the pursued. If on the other hand switchover did not take place, i.e. $\tau_i = T$, then the game of pursuit is terminated with the arrival of the vector $B_i(t) z_i^{(1)}(t)$ at M_i. Here the vector function $z_i^{(1)}(\cdot)$ and $z_i^{(2)}(\cdot)$ represent, respectively, the solutions of systems (1) and (2).

This is then the formulation of the problem of pursuit which we will consider. Naturally, the case when the pursuit by the method considered below is impossible for all $\tau_i = 0$ (or $\tau_i = +\infty$ for $\theta = +\infty$) simultaneously is of interest. We consider the case when pursuit is possible at all τ_i simultaneously equal to 0 or $+\infty$. Although it also follows from the assertions made below, it was already given in /3/. We assume that precisely such a case was realized in the game (1), (2).

Let us write $N = \{1, \ldots, m\}$, $\pi_i\colon R^{m_i} \to L_i^1$, $z^\circ = \{z_1^\circ, \ldots, z_m^\circ\}$. Suppose also $\Omega_i^{(1)}(t, \tau)$ and $\Omega_i^{(2)}(t, \tau)$ are the matrizants of the systems $z_i^{\cdot} = C_i^{(1)}(t) z_i$ and $z_i^{\cdot} = C_i^{(2)}(t) z_i$ respectively, with initial conditions

$$\Omega_i^{(j)}(\tau, \tau) = E \;\; (j = 1, 2; \; i = 1, \ldots, m) \;\; D_i^{(1)}(t, \tau_i, s) =$$
$$\pi_i \Omega_i^{(2)}(t, \tau_i) B_i(\tau_i) \Omega_i^{(1)}(\tau_i, s) \;\; D_i^{(2)}(t, \tau_i, s) = \pi_i \Omega_i^{(2)}(t, s)$$

Assumption 1. Matrices $A_i^{(j)}(t)$ $(i = 1, \ldots, m; j = 1, 2)$ depending continuously on time exist such, that for $t \geqslant 0, 0 \leqslant \tau_i \leqslant t, \tau_i \leqslant \theta$ the following sets are non-empty:

$$W_i(t, \tau_i, s) = D_i^{(1)}(t, \tau_i, s) f_i^{(1)}(P_i^{'1)}) \stackrel{*}{-}$$
$$A_i^{(1)}(\tau_i - s) D_i^{'(1)}(t, \tau_i, s) g_i^{(1)}(Q) \neq \varnothing, \quad s \in [0, \tau_i)$$
$$W_i(t, \tau_i, s) = D_i^{'2)}(t, \tau_i, s) f_i^{'2)}(P_i^{(2)}) \stackrel{*}{-}$$
$$A_i^{'2)}(t - s) D_i^{(2)}(t, \tau_i, s) g_i^{(2)}(Q) \neq \varnothing, s \in [\tau_i, +\infty)$$
$$M_i^3(t, \tau_i) = M_i^2 \stackrel{*}{-} \Big[\int_0^{\tau_i} (A_i^{(1)}(\tau_i - s) - E) D_i^{'(1)}(t, \tau_i, s) g_i^{(1)}(Q)\, ds +$$
$$\int_{\tau_i}^{t} (A_i^{(2)}(t - s) - E) D_i^{'(2)}(t, \tau_i, s) g_i^{(2)}(Q)\, ds\Big] \neq \varnothing$$

as well as the functions $\gamma_i(t, \tau_i, s)\colon \gamma_i(t, \tau_i, s) \in W_i(t, \tau_i, s)$, $s \in [0, t]$ semicontinuous in s from above.

We write

$$\lambda_i(t, \tau_i, s, v, z_i^\circ, m_i^3) = \max\Big\{\lambda, \lambda \geqslant 0, -\lambda \times \Big[D_i^{(1)}(t, \tau_i, 0) z_i^\circ - m_i^3 + \int_0^{\tau_i} \gamma_i(t, \tau_i, s)\, ds\Big] \in$$
$$D_i^{(1)}(t, \tau_i, s) f_i^{(1)}(P_i^{(1)}) - A_i^{(1)}(\tau_i - s) D_i^{(1)}(t, \tau_i, s) g_i^{(1)}(v) -$$
$$\gamma_i(t, \tau_i, s)\Big\}, \quad s \in [0, \tau_i)$$
$$\lambda_i(t, \tau_i, s, v, z_i^\circ, m_i^3) = \max\Big\{\lambda, \lambda \geqslant 0, -\lambda \times$$
$$\Big[D_i^{(1)}(t, \tau_i, 0) z_i^\circ - m_i^3 + \int_{\tau_i}^{t} \gamma_i(t, \tau_i, s)\, ds\Big] \in$$
$$D_i^{(2)}(t, \tau_i, s) f_i^{(2)}(P_i^{(2)}) - A_i^{(2)}(t - s) D_i^{(2)}(t, \tau_i, s) g_i^{(2)}(v),$$
$$s \in [\tau_i, +\infty), \quad m_i^3 \in M_i^3(t, \tau_i)$$
$$\lambda_i(t, \tau_i, s, v, z_i^\circ, M_i^3(t, \tau_i)) = \max_{m_i^3 \in M_i^3(t, \tau_i)} \lambda_i(t, \tau_i, s, v, z_i^c, m_i^3),$$
$$s \in [0, t], \quad i = 1, \ldots, m$$
$$\rho(t, \tau_1, \ldots, \tau_m, z^\circ) = \inf_{v(\cdot)} \max_{i \in N} \int_0^t \lambda_i(t, \tau_i, s, v(s), z_i^\circ, M_i^3(t, \tau_i))\, ds$$

Assumption 2. There exist t°, τ_i° $(i = 1, \ldots, m)$, $0 \leqslant \tau_i^\circ \leqslant t^\circ$, $\tau_i^\circ \leqslant \theta$, such that $\rho(t^\circ, \tau_1^\circ, \ldots, \tau_m^\circ, z^\circ) = 1$.

Theorem. Let the assumptions 1 and 2 hold for the game (1), (2) with initial positions z°. Then the game of pursuit has a solution and t° is the guaranteed time of its termination.

Proof. Let us consider the function $\lambda_i(t, \tau_i, s, v, z_i^\circ, m_i^3)$. The function is semicontinuous when i and z_i° are fixed, from above, over the set of arguments t, τ_i, s, v, m_i^3. Therefore the multivalued mapping $G_i^3(t, \tau_i, s, v) = \{m_i^3 \in M_i^3(t, \tau_i): \lambda_i(t, \tau_i, s, v, z_i^\circ, m_i^3) = \lambda_i(t, \tau_i, s, v, z_i^\circ, M_i^3(t, \tau_i)\}$ is semicontinuous from above with respect to the inclusions, and by virtue of the known property of multivalued mappings there exists a Borel-measurable selector /3/ $m_i^3(t, \tau_i, s, v) \in G_i^3(t, \tau_i, s, v)$.

Let $v(s)$ be an arbitrary, admissible control of the pursued. We shall construct the control of the i-th pursuer as follows:

1) up to the switchover $0 \leqslant s \leqslant \tau_i^\circ \leqslant t^\circ$, provided that at the instant s

$$\rho_i(s, v(\xi), 0 \leqslant \xi \leqslant s) = \int_0^{s_i} \lambda_i(t^\circ, \tau_i^\circ, \xi, v(\xi), z_i^\circ, m_i^3(t^\circ, \tau_i^\circ, \xi, v(\xi)))\, d\xi < 1$$

the function $u_i^{(1)}(s) \in P_i^{(1)}$ is a solution of the equation

$$\begin{aligned} &-\lambda_i(t^\circ, \tau_i^\circ, s, v(s), z_i^\circ, M_i^3(t^\circ, \tau_i^\circ))\Big(D_i^{(1)}(t^\circ, \tau_i^\circ, 0)\, z_i^\circ - \\ &m_i^3(t^\circ, \tau_i^\circ, s, v(s)) + \int_0^{\tau_i^\circ} \gamma_i(t^\circ, \tau_i^\circ, \xi)\, d\xi\Big) = \\ &D_i^{(1)}(t^\circ, \tau_i^\circ, s)\, f_i^{(1)}(u_i^{(1)}(s)) - A_i^{(1)}(\tau_i^\circ - s)\, D_i^{(1)}(t^\circ, \tau_i^\circ, s)\, g_i^{(1)}(v(s)) - \gamma_i(t^\circ, \tau_i^\circ, s) \end{aligned} \tag{3}$$

if s_i^1 is the first instant, when $\rho_i(s_i^1, v(\xi), 0 \leqslant \xi \leqslant s_i^1) = 1$, then at $s \in (s_i^1, \tau_i^\circ)$ $u_i^1(s)$ is a solution of the equation

$$D_i^{(1)}(t^\circ, \tau_i^\circ, s)\, f_i^{(1)}(u_i^{(1)}(s)) - A_i^{(1)}(\tau_i^\circ - s)\, D_i^{(1)}(t^\circ, \tau_i^\circ, s)\, g_i^{(1)}(v(s)) = -\gamma_i(t^\circ, \tau_i^\circ, s) \tag{4}$$

2) after the switchover $0 \leqslant \tau_i^\circ \leqslant s \leqslant t^\circ$. Similarly, if for the given s $\rho_i(s, v(\xi), 0 \leqslant \xi < s) \leqslant 1$, then the function $u_i^{(2)}(s) \in P_i^{(2)}$ will be chosen as a solution of the equation

$$\begin{aligned} &-\lambda_i(t^\circ, \tau_i^\circ, s, v(s), z_i^\circ, M_i^3(t^\circ, \tau_i^\circ))\,(D_i^{(1)}(t^\circ, \tau_i^\circ, 0)\, z_i^\circ - m_i^3(t^\circ, \tau_i^\circ, s, v(s)) + \\ &\int_{\tau_i^\circ}^{t^\circ} \gamma_i(t^\circ, \tau_i^\circ, \xi)\, d\xi) = D_i^{(2)}(t^\circ, \tau_i^\circ, s)\, f_i^{(2)}(u_i^{(2)}(s)) - \\ &A_i^{(2)}(t^\circ - s)\, D_i^{(2)}(t^\circ, \tau_i^\circ, s)\, g_i^{(2)}(v(s)) - \gamma_i(t^\circ, \tau_i^\circ, s) \end{aligned} \tag{5}$$

if s_i^1 is the first instant, when $\rho_i(s_i^1, v(\xi), 0 \leqslant \xi \leqslant s_i^1) = 1$, then for $s \in (s_i^1, t^\circ]$ $u_i^{(2)}(s)$ is a solution of the equation

$$D_i^{(2)}(t^\circ, \tau_i^\circ, s)\, f_i^{(2)}(u_i^{(2)}(s)) - A_i^{(2)}(t^\circ - s)\, D_i^{(2)}(t^\circ, \tau_i^\circ, s)\, g_i^{(2)}(v(s)) = \gamma_i(t^\circ, \tau_i^\circ, s) \tag{6}$$

According to the A.F. Filippov theorem the functions $u_i^{(j)}(t) = U_i^{(j)}(s, z_i^\circ, v(s))$ $(j = 1, 2)$ constructed will be Lebesgue-measurable. In addition, the construction yields the inclusion $u_i^{(j)} \in P_i^{(j)}$ $(j = 1, 2)$. In this manner we find the strategy $(\tau_i^\circ, U_i^{(1)}, U_i^{(2)})$ $(i = 1, \ldots, m)$ of the i-th pursuer. The strategies will guarantee the termination of the pursuit at the instant t°. Indeed, according to assumption 2, for the given control $v(s), s \in [0, t^\circ]$ of the pursued a number $k \in N$ and an instant of time s_k, $0 \leqslant s_k \leqslant t^\circ$ exist such that $\rho_k(s_k, v(\xi), 0 \leqslant \xi \leqslant s_k) = 1$. According to the Cauchy formula we have, for k, (see (3)–(6))

$$\begin{aligned} &\pi_k z_k(t^\circ) = \pi_k \Omega_k^{(2)}(t^\circ, \tau_k^\circ)\, B_k(\tau_k^\circ)\, \Big\{\Omega_k^{(1)}(\tau_k^\circ, 0)\, z_k^\circ + \\ &\int_0^{\tau_k^\circ} [\Omega_k^{(1)}(\tau_k^\circ, s)\, f_k^{(1)}(u_k^{(1)}(s)) - \Omega_k^{(1)}(\tau_k^\circ, s)\, g_k^{(1)}(v(s))]\, ds\Big\} + \\ &\int_{\tau_k^\circ}^{t^\circ} [\pi_k \Omega_k^{(2)}(t^\circ, s)\, f_k^{(2)}(u_k^{(2)}(s)) - \pi_k \Omega_k^{(2)}(t^\circ, s)\, g_k^{(2)}(v(s))]\, ds = \\ &\Big[D_k^{(1)}(t^\circ, \tau_k^\circ, 0)\, z_k^\circ + \int_0^{t^\circ} \gamma_k(t^\circ, \tau_k^\circ, s)\, ds\Big] \times \\ &\Big[1 - \int_0^{s_k} \lambda_k(t^\circ, \tau_k^\circ, s, v(s), z_k^\circ, m_k^3(t^\circ, \tau_k^\circ, s, v(s))\, ds\Big] + \\ &\int_0^{s_k} \lambda_k(t^\circ, \tau_k^\circ, s, v(s), z_k^\circ, m_k^3(t^\circ, \tau_k^\circ, s, v(s))\, m_k^3(t^\circ, \tau_k^\circ, s, v(s))\, ds + \\ &\int_0^{\tau_k^\circ} (A_k^{(1)}(\tau_k^\circ - s) - E)\, D_k^{(1)}(t^\circ, \tau_k^\circ, s)\, g_k^{(1)}(v(s))\, ds + \\ &\int_{\tau_k^\circ}^{t^\circ} (A_k^{(2)}(t^\circ - s) - E)\, D_k^{(2)}(t^\circ, \tau_k^\circ, s)\, g_k^{(2)}(v(s))\, ds \in M_k^3 \end{aligned}$$

Thus the k-th pursuer captures the pursued at the instant t^0 The theorem is proved.

Example. Let us consider the following differential game of group pursuit with a variable structure. Up to the instant of switchover the equations of motion of the m pursuers and a single pursued have the form

$$x_i^{\cdot\cdot} + \alpha_i x_i^{\cdot} = u_i,\ i = 1, \ldots, m;\ y^{\cdot} = v,\ t \in (0, \tau_i) \tag{7}$$

and after the switchover,

$$x_i^{\cdot} = u_i,\ i = 1, \ldots, m;\ y^{\cdot} = v,\ t \in (\tau_i, +\infty) \tag{8}$$

The initial values are $x_i(0) = x_i^0$, $y(0) = y^0$; $0 < \alpha < 1$. The pursuers must go through the switchover by the time $\theta > 0$ inclusive. The game of pursuit will be terminated at some finite instant of time when the condition $\|x_i - y\| \leqslant R$, $R = -\alpha^{-1} + \alpha^{-2}(\alpha - 1)\ln(1 - \alpha)$ is satisfied for at least one i.

Here $x_i,\ y,\ u_i,\ v \in R^2$; $\|u_i\| \leqslant 1$, $\|v\| \leqslant 1$. The substitution $z_i = (z_{i1}, z_{i2}) = (x_i - y, x_i^{\cdot})$ in (7) and $z_i = x_i - y$ in (8) leads to the following differential game:

$$z_{i1}^{\cdot} = z_{i2} - v \tag{9}$$

$$z_{i2}^{\cdot} = -\alpha z_{i2} + u_i,\ i = 1, \ldots, m,\ t \in (0, \tau_i)$$

$$z_i = u_i - v,\ i = 1, \ldots, m,\ t \in (\tau_i, +\infty) \tag{10}$$

The initial conditions are

$$z_{i1}(0) = z_{i1}^0,\ z_{i2}(0) = z_{i2}^0;\ \pi_i = E$$

$$B_i(\tau_i) \equiv \begin{Vmatrix} E & 0 \\ 0 & 0 \end{Vmatrix},\quad C_i^{(1)}(t) \equiv \begin{Vmatrix} 0 & E \\ 0 & -\alpha E \end{Vmatrix},\quad C_i^{(2)}(t) \equiv 0$$

$$P_i^{(1)} = P_i^{(2)} = Q = S_1(0) = \{\xi \in R^2 : \|\xi\| \leqslant 1\}$$

Here E is the unit matrix and O is the null 2×2 matrix.

For this game we have

$$M_i = S_R(0)$$

$$\Omega_i^{(1)}(t, \tau) = \exp\{(t - \tau) C_i^{(1)}\} = \begin{Vmatrix} E & \alpha^{-1}(1 - \exp(-\alpha(t - \tau)))E \\ 0 & E - \alpha t E + (\alpha^2 t^2/2!)E - \ldots \end{Vmatrix}$$

$$\Omega_i^{(2)}(t, \tau) = \exp\{(t - \tau) C_i^{(2)}\} = E \quad (i = 1, \ldots, m)$$

Assumption 1 will hold for the game (9), (10) if we write $A_i^{(1)}(t) = \mu_i(t) E$, $A_i^{(2)}(t) \equiv E$, where

$$\mu_i(t) = \begin{cases} \alpha^{-1}(1 - e^{-\alpha t}), & 0 \leqslant t \leqslant -\alpha^{-1}\ln(1 - \alpha) \\ 1, & t > -\alpha^{-1}\ln(1 - \alpha) \end{cases}$$

Moreover we have

$$\lambda_i(t, \tau_i, s, v, z_i^0, m_i^2) = \|\eta_i(\tau_i)\|^{-1} \mu_i(t - \tau_i)\{\omega_i(v, \tau_i) + ((\omega_i(v, \tau_i))^2 + (\alpha\mu_i(\tau_i - s))^{-2}(1 - \exp(-\alpha(\tau_i - s)))^2 - \|v\|^2)^{1/2}\},\ s \in [0, \tau_i)$$

$$\lambda_i(t, \tau_i, s, v, z_i^0, m_i^2) = \|\eta_i(\tau_i)\|^{-1}\{\omega_i(v, \tau_i) + ((\omega_i(v, \tau_i))^2 + 1 - \|v\|^2)^{1/2}\},\ s \in [\tau_i, +\infty)$$

$$\eta_i(\tau_i) = z_{i1}^0 + \alpha^{-1}(1 - e^{-\alpha\tau_i}) z_{i2}^0 - m_i^2,\ \omega_i(v, \tau_i) = (v, \|\eta_i(\tau_i)\|^{-1}\eta_i(\tau_i))$$

We have the following estimate for the function $\lambda_i(t, \tau_i, s, v, z_i^0, m_i^2)$ prior to the switchover

$$0 \leqslant \int_0^{\tau_i} \lambda_i(t, \tau_i, s, v(s), z_i^0, m_i^2)\, ds \leqslant \tag{11}$$

$$\tau_i(\alpha\|\eta_i(\tau_i)\|)^{-1}(\alpha + \sqrt{\alpha^2 + 1}) \leqslant \theta(\alpha\|\eta_i(\tau_i)\|)^{-1}(\alpha + \sqrt{\alpha^2 + 1})$$

For the function $\lambda_i(t, \tau_i, s, v(s), z_i^2, m_i^2)$ we find, after the switchover, that if the condition

$$0 \in \operatorname{int}\operatorname{co}\bigcup_{i=1}^{m}(z_{i1}^0 + \alpha^{-1}(1 - e^{-\alpha\tau_i}) z_{i2}^0 - M_i^2(t, \tau_i)) \tag{12}$$

is satisfied then the condition

$$\inf_{v(\cdot)} \max_{i \in N} \int_{\tau_i}^{T} \lambda_i(t, \tau_i, s, v(s), z_i^0, M_i^2(t, \tau_i))\, ds \geqslant A > 0$$

is satisfied for any previously specified number $A > 0$, provided that a sufficiently large T is taken.

Let $m = 3$. We write $\theta = 0{,}1R(\alpha + \sqrt{\alpha^2 + 1})^{-1}\alpha$ and use as an example the following initial conditions:

$$z_{11}^0 = (-2R, R),\quad z_{21}^0 = (0, 2R),\quad z_{31}^0 = (2R, R),\quad z_{12}^0 = (0, -2R(1 - e^{-\lambda\tau})^{-1}\alpha),\quad z_{22}^0 = (0, 0),\quad z_{32}^0 = z_{12}^0,\quad 0 \leqslant \tau \leqslant \theta$$

We can easily see that for these initial conditions, condition (12) holds for $\tau_i = \tau$ $(i = 1, 2, 3)$. Moreover we obtain the following estimate for (11):

$$\int_0^{\tau_i} \lambda_i(t, \tau_i, s, v(s), z_i^*, m_i^s)\, ds < 1, \quad \forall \tau_i \in [0, \theta], \quad i = 1, 2, 3$$

Therefore the capture of the pursued is possible in the present game only when the instant of switchover $\tau_i \in (0, \theta]$, e.g. $\tau_i = \tau\ (i = 1, 2, 3)$.

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ON CONSTRUCTING A FUNCTIONAL IN THE PROBLEM OF OPTIMAL CONTROL*

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The problem of constructing a functional in the theory of control of material systems is considered as an inverse problem of dynamics /1/. It was A.M. Letov /2/ who first became aware of the practical value of the inverse problems in optimal control. He solved a number of inverse problems of choosing the optimal functional in problems of controlling aircraft. The approach was also successfully used in problems of robotics /3/. The procedure used in solving inverse problems makes it possible to combine the merits and virtues of the engineering problems based on formulating the control laws from the conditions of motion according to a given program, with the possibilities offered by methods of optimal control theory.

Let us consider a controlled object described by a system of ordinary differential equations

$$x^{\cdot} = f(x, u, t), \quad x \in R^n, \quad u \in R^r \tag{1}$$

where f is a continuously differentiable vector function, u is a piecewise continuous control and $0 \leqslant t \leqslant T$. The initial condition $a = x(0)$ and time T will be assumed given. We shall call the pair $\{u(t), x(t)\}$ the admissible process, if $u(t), x(t)$ satisfy (1).

We shall treat the inverse problem of optimal control for the object in question as a problem of determining a continuously differentiable function $f_0(x, u)$, such that the solution of the problem of maximizing the functional

$$J = \int_0^T f_0(x, u)\, dt \tag{2}$$

leads to the given admissible process $\{u^*(t), x^*(t)\}$.

Let us write the unknown function $f_0(x, u)$ as the sum of the continuously differentiable functions $\varphi_s(x, u)\ (s = 1, 2, \ldots, p)$ in the form

$$f_0(x, u) = \sum_{s=1}^{p} c_s \varphi_s(x, u)$$

In this case the inverse problem of optimal control will be reduced to finding all coefficients $c_s\ (s = 1, 2, \ldots, p)$, such that the maximization of the functional

$$J = \int_0^T \sum_{s=1}^{p} c_s \varphi_s(x, u)\, dt \tag{3}$$

when (1) is satisfied, leads to the given admissible process $\{u^*(t), x^*(t)\}$, $0 \leqslant t \leqslant T$.

In order to exclude the trivial case where all coefficients c_s are zero (in this case any admissible process will be optimal for a functional identically equal to zero), we shall introduce the concept of a non-degenerate solution of the inverse problem: we will call the solution of the inverse problem non-degenerate, if at least one of the coefficients c_s is not

*Prikl.Matem.Mekhan., 50, 1, 159-163, 1986